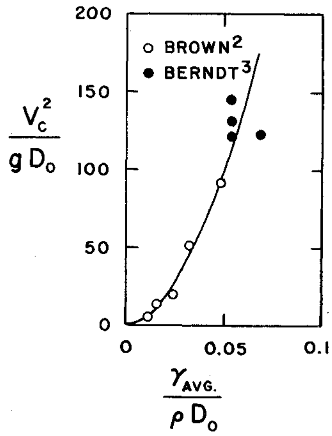

Fig. 2 v_c^2/gD_0 vs $\gamma_{avg}/\rho D_0$.

Figure 2 shows a plot of v_c^2/gD_0 vs $\gamma_{avg}/\rho D_0$ based on the data of Table 1. The equation of the curve in Fig. 2 is

$$v_c^2/gD_0 = (2.74 \times 10^4)(\gamma_{avg}/\rho D_0)^{1.87} \tag{3}$$

Equation (3) indicates that v_c^2 varies approximately directly with $g\gamma_{avg}^2/\rho^2 D_0$, which agrees with Brown's observation[2] that v_c varies approximately as $D_0^{-1/2}$.

As seen, the data of Fig. 2 are reasonably well correlated. However, whether the correlation is real or merely fortuitous remains open to question, since the two sets of data in Table 1 do not meet the constraint of geometrical similarity. This Note is therefore presented in the hope of stimulating further research and experimentation on parachute critical opening velocity.

References

[1] O'Hara, F., "Notes on the Opening Behaviour and the Opening Forces of Parachutes," *Journal of the Royal Aeronautical Society*, Vol. 53, Nov. 1949, pp. 1053–1062.

[2] Brown, W. D., *Parachutes*, Pitman & Sons, London, 1951, pp. 66–86.

[3] Berndt, R. J., "Experimental Determination of Parameters for the Calculation of Parachute Filling Times," *Jahrbuch 1964 der WGLR*, Vieweg & Sohn, Braunschweig, West Germany, 1965, pp. 299–316.

[4] French, K. E., "The Initial Phase of Parachute Inflation," *Journal of Aircraft*, Vol. 6, No. 4, July-August 1969, pp. 376–378.

Coriolis Coupled Bending Vibrations of Hingeless Helicopter Rotor Blades

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Nomenclature

$(EI)_v$ = in-plane flexural rigidity, lb-ft^2
$(EI)_w$ = out-of-plane flexural rigidity, lb-ft^2
L = lift per unit length of blade, lb/ft
Q = axial tension force, lb
R = radius of rotor, span of rotor blade, ft
V = in-plane blade displacement, ft
W = out-of-plane blade displacement, ft
a_i,b_i = coefficients in Galerkin series expansion
a_v,a_w = absolute acceleration components in-plane and out-of-plane, respectively, ft/sec^2

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m = mass per unit length, slug/ft
r = radial position along blade, ft
t = time, sec
v = amplitude of in-plane displacement, ft
w = amplitude of out-of-plane displacement
x = dimensionless blade span coordinate
β = preconing angle of blade with respect to plane of rotation, rad
γ = coupling factor in characteristic equation
ξ = dummy variable of integration, ft
η = dimensionless dummy variable of integration
ψ = blade azimuth coordinate, rad
ν = natural frequency ratio, cycles/revolution
Ω = frequency of steady rotation of blade, rad/sec
$'$ = differentiation with respect to span coordinate or dimensionless span coordinate
$\cdot$ = differentiation with respect to time

Introduction

The bending vibrations of conventional articulated helicopter rotor blades are usually considered to be uncoupled oscillations, involving small displacements normal to or in the plane of steady rotation. The resulting modes of vibration are similar to those obtained for a nonuniform, pin-free beam, modified by the additional inertia loading of the centrifugal force field. In the case of the hingeless or cantilever rotor blade, it is necessary to utilize an initial "preconing" or built-in root slope to minimize the steady bending moment caused by the lift loading of the blade by counteracting it with one caused by the centrifugal force. This preconing results in a Coriolis inertia loading which couples the out-of-plane and in-plane bending vibrations, leading in some cases of practical interest to significant differences in the coupled natural frequencies of vibration relative to those predicted by the usual uncoupled vibration analysis.

Analysis

For the small oscillations of interest, we may utilize the equation of equilibrium of a beam column,[1] modified to account for the varying axial tension along the span caused by centrifugal force and the appropriate transverse inertia and lift loadings. The equations of equilibrium for small blade motions normal to the plane of rotation and in the plane of rotation, respectively, are

$$[(EI)_w W'']'' - (QW')' + ma_w = L(r,t) \tag{1}$$

$$[(EI)_v V'']'' - (QV')' + ma_v = 0 \tag{2}$$

$$Q = \Omega^2 \int_r^R m(\xi)\xi d\xi \tag{3}$$

$$a_w = \ddot{w} + 2\Omega\dot{v}\beta + \Omega^2 r\beta \tag{4}$$

$$a_v = \ddot{v} - \Omega^2 v - 2\Omega\dot{w}\beta \tag{5}$$

Equation (1) corresponds to the more familiar one for the out-of-plane vibration,[2] except for the Coriolis coupling term and the transverse loading caused by the small preconing angle β and centrifugal force. The latter effect is considered by assuming that β has been selected such that

$$\Omega^2 r\beta = L_{steady} \tag{6}$$

This is a good approximation at the normal design loading where the steady lift loading is virtually a linear function of span except at the most outboard blade stations due to the blade trailing tip vortex.

The reduced equations of motion are now solved for the modes of free vibration by first nondimensionalizing and then separating[3] the coupled partial differential equations.

Let $r = Rx$, $\psi = \Omega t$, and

$$W(x,\psi) = w(x)\sin\nu\psi \tag{7}$$

$$V(x,\psi) = v(x)\cos\nu\psi \tag{8}$$

$$L(x,\psi) = L_{steady} = \Omega^2 Rx\beta \tag{9}$$

Note that the Coriolis coupling results in a 90° phase shift between the w and v components of displacement. The separated equations for free vibration then become

$$\left[\frac{(EI)_w}{\Omega^2 R^4} w''\right]'' - \left[\left(\int_x^1 m\eta d\eta\right) w'\right]' - m(\nu^2 w + 2\nu\beta v) = 0 \quad (10)$$

$$\left[\frac{(EI)_v}{\Omega^2 R^4} v''\right]'' - \left[\left(\int_x^1 m\eta d\eta\right) v'\right]' - m[(\nu^2 + 1)v + 2\nu\beta w] = 0 \quad (11)$$

The appropriate boundary conditions are the same as for the uncoupled case of a cantilever blade

$$v(0) = w(0) = v'(0) = w'(0) = v''(1) = w''(1) = v'''(1) = w'''(1) = 0 \quad (12)$$

Equations (10) and (11) are now solved by Galerkin's method,[4] utilizing the known, uncoupled modes of vibration which are also the solutions for no preconing or vanishing β. This permits a direct comparison and evaluation of the significance of the Coriolis coupling. Let

$$w(x) \cong \sum_{i=1}^{n} a_i w_i(x) \quad (13)$$

$$v(x) \cong \sum_{i=1}^{n} b_i v_i(x) \quad (14)$$

Associated with w_i and v_i, the ith modes of uncoupled out-of-plane and in-plane vibrations are the uncoupled natural frequency ratios ν_{wi} and ν_{vi}, respectively. Substituting Eqs. (13) and (14) into Eqs. (10) and (11), multiplying through by w_j and v_j, respectively, and integrating over the dimensionless span there results the $2n$ homogeneous algebraic equations

$$\int_0^1 mw_j \sum_{i=1}^{n} [a_i(\nu^2 - \nu_{wi}^2)w_i + 2\nu\beta b_i v_i]dx = 0, \quad j = 1,2,\ldots,n \quad (15)$$

$$\int_0^1 mv_j \sum_{i=1}^{n} [b_i(\nu^2 - \nu_{vi}^2)v_i + 2\nu\beta a_i w_i]dx = 0, \quad j = 1,2,\ldots,n \quad (16)$$

Limiting our attention in this analysis to the fundamental modes of coupled vibration, the series expansions in the Galerkin solution are truncated to

$$w \cong a_1 w_1 \text{ and } v \cong b_1 v_1 \quad (17)$$

This results in the biquadratic characteristic equation

$$\left(\frac{\nu}{\nu_{w1}}\right)^4 - \left[1 + \left(\frac{\nu_{v1}}{\nu_{w1}}\right)^2 + \left(\frac{2\beta\gamma}{\nu_{w1}}\right)^2\right]\left(\frac{\nu}{\nu_{w1}}\right)^2 + \left(\frac{\nu_{v1}}{\nu_{w1}}\right)^2 = 0 \quad (18)$$

$$\gamma^2 = \left(\int_0^{'} mw_1v_1dx\right)^2 \Big/ \left(\int_0^{'} mw_1^2dx\right)\left(\int_0^{'} mv_1^2dx\right) \quad (19)$$

where the coupled frequency ratio ν is normalized by the uncoupled out-of-plane frequency ratio ν_{w1}, and is seen to depend on two parameters: first, the ratio of the in-plane to out-of-plane fundamental uncoupled bending frequencies and a modal coupling factor proportional to the preconing angle.

Sample Calculation

To illustrate the potential importance of the Coriolis coupling, a sample calculation has been carried out for a typical fundamental uncoupled out-of-plane frequency ratio[5] $\nu_{w1} = 1.11$ and an uncoupled in-plane frequency ratio of $\nu_{v1} = 0.70$ which is typical of a helicopter designed with in-plane flexural rigidity close to that for out-of-plane bending.[6] The precone angle β is taken as 6°, corresponding to a heavy aerodynamic loading from Eq. (6). The modal coupling factor γ is approximated as unity. This results in the coupled frequency ratios $\nu = 1.14$ and $\nu = 0.68$.

Concluding Remarks

The relatively small numerical differences between the coupled and uncoupled frequencies in the sample calculation can be significant in the flying qualities and ground resonance stability of the helicopter. In the case of the higher frequency (dominantly out-of-plane bending) the increase in frequency caused by Coriolis coupling will tend to increase rotor control power.[5] In the case of the lower frequency (dominantly inplane bending), the dynamic stability of the system on the ground ("ground resonance") is sensitive to small changes.[6] Perhaps of equal importance in this case is that the out-of-plane component caused by Coriolis coupling contributes some aerodynamic coupling which should be included in the ground resonance analysis of hingeless rotor helicopters.

References

[1] Hoff, N. J., *The Analysis of Structures*, Wiley, New York, 1956, pp. 151–158.

[2] Bisplinghoff, R. L., Ashley, H., and Halfman, R. L., *Aeroelasticity*, Addison-Wesley, Reading, Mass., 1955, pp. 95–98.

[3] Karman, T. V. and Biot, M. A., *Mathematical Methods in Engineering*, McGraw-Hill, New York, 1940, pp. 283–290.

[4] Hurty, W. C. and Rubinstein, M. F., *Dynamics of Structures*, Prentice-Hall, Englewood Cliffs, N. J., 1964, pp. 186, 243–245.

[5] Young, M. I., "A Simplified Theory of Hingeless Rotors with Application to Tandem Helicopters," *Proceedings of The Annual Forum of The American Helicopter Society*, 1962, pp. 38–44.

[6] Lytwyn, R. T., Miao, W., and Woitsch, W., "Airborne and Ground Resonance of Hingeless Rotors," *Journal of The American Helicopter Society*, Vol. 16, April 1971, pp. 2–9.

Kernel Function for Nonplanar Oscillating Surfaces in Supersonic Flow

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RODEMICH[1] and Landahl[2] have given the subsonic acceleration potential kernel for nonplanar configurations in a form that has been rewritten by Rodden, Giesing, and Kalman[3] as

$$K = \exp(-i\omega x_0/U)(K_1T_1/r^2 + K_2T_2^*/r^4) \quad (1)$$

where ω is the frequency, x_0 is the distance between the sending and receiving points parallel to the freestream, U is the velocity of the freestream,

$$T_1 = \cos(\gamma_r - \gamma_s) \quad (2)$$

$$T_2^* = (z_0 \cos\gamma_r - y_0 \sin\gamma_r)(z_0 \cos\gamma_s - y_0 \sin\gamma_s) \quad (3)$$

$$r = (y_0^2 + z_0^2)^{1/2} \quad (4)$$

y_0 and z_0 are the Cartesian distances perpendicular to the freestream between the sending and receiving points, and γ_r and

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